To compute $(-1729) \times (343)$ on a calculator, PRESS

[(−)] [1729] [×] [343] [=] which should show -593047.

Whole numbers are called **integers**. There are numbers between whole numbers which will be discussed in subsequent sections.

Fig. 4

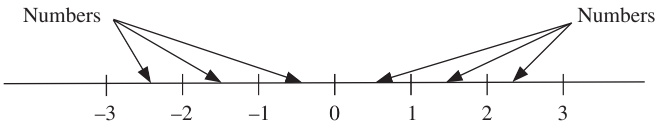

Figure 4 shows a portion of the real number line indicating some of the whole numbers. Any number on this line is a **real number**.

SUMMARY

The symbols $<$ and $\leq$ mean less than and less than or equal to respectively. Similarly $>$ and $\geq$ mean greater than and greater than or equal to respectively.

For adding and subtracting, remember the following rules:

Adding a negative number is the same as subtracting a positive number.

Subtracting a negative number is the same as adding a positive number.

For division and multiplication:

If both numbers have the same sign then the result is positive, otherwise it is negative.

Exercise **Intro(a)**

Solutions at end of book. Complete solutions available at www.palgrave.com/science/engineering/singh

1 Fill in the most appropriate symbol $<$, $\leq$, $>$ or $\geq$ in place of □ for the following:

a 17 □ -17 **b** -17 □ 17
c -2 □ -1 **d** -5 □ -5
e -2 □ 0

2 Complete the correct symbol, $+$, $-$, $\div$, $\times$, in place of □ for the following:

a 3 □ $2 = 5$ **b** 3 □ $-2 = 5$
c 9 □ $3 = 3$ **d** -7 □ $(-11) = 77$
e -6 □ $(-5) = -1$

3 Evaluate the following:

a $7 - 12$ **b** $-3 + 1$
c $-3 - (-1)$ **d** $-11 + (-11)$
e $-11 - (-11)$

4 Calculate

a $(-6) \div (-2)$ **b** $(-6) \times 2$
c $(-6) \times (-2)$ **d** $(-6) \div 2$
e $(-6) \div (-2)$ **f** $6 \div (-2)$
g $(-1) \times (-2) \times (-8)$

5 By using your calculator, or otherwise, compute the following:

a $(-343) \times 343$
b $(-343) \times (-343)$
c $(-729) \div 81$
d $\dfrac{(-729)}{81}$
e $(-666) - (-1945)$
f $(-2) \times (-5) \times (-7) \times (-10)$

SECTION B Indices

By the end of this section you will be able to:

- understand the terms **index**, **power** and **indices**
- evaluate indices
- evaluate square roots, cube roots etc.

B1 Powers and roots

Instead of writing $\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{5 copies}}$, can you remember a shorter way of writing this number?

It can be written as 2 with superscript 5, 2^5. The superscript is called the index or the power, so in 2^5 the 5 is the **power** or **index**. The plural of index is **indices**. We can write 7×7 as 7^2 pronounced '7 squared', that is

$$7 \times 7 = 7^2$$

How can we write 7 × 7 × 7 in this index format?

$$\underbrace{7 \times 7 \times 7}_{\text{3 copies}} = 7^3$$

7^3 is pronounced '7 cubed'. We can write repetitive multiplication of the same number with an index (or power).

What is 9^7 (9 to the index 7) equal to?

$$\underbrace{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9}_{\text{7 copies}}$$

To evaluate this we use a calculator. PRESS

[9] [x^y] [7] [=], which should show 4782969. Therefore

$$9^7 = 4782969$$

On some calculators you might have to press the [∧] key instead of the [x^y] key.

See the handbook of your calculator.

Powers of numbers can be easily evaluated on a calculator. Let's do an example.

Example 4

By using your calculator, or otherwise, compute the following:

a 3^4 **b** 88^2 **c** 20^3 **d** 20^1 **e** 7^0 **f** $(-2)^3$

Solution

Use the x^y or ∧ button on your calculator.

a $3^4 = \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 copies}} = 81$

Example 4 *continued*

b $88^2 = 88 \times 88 = 7744$
c $20^3 = 20 \times 20 \times 20 = 8000$

d **How can we write 20^1?**

$$20^1 = 20$$

Any number to the index 1 is the number itself because it is **not** multiplied by itself again.

e Use your calculator to find 7^0:

$$7^0 = 1$$

There is a general result which says that: any number, apart from zero, to the index 0 gives 1.

f Odd number of negatives multiplied gives a negative number:

$$\begin{aligned}(-2)^3 &= \underbrace{(-2) \times (-2)}_{=4} \times (-2)\\ &= 4 \times (-2)\\ &= -8\end{aligned}$$

Since $9^2 = 81$, can we somehow extract 9 from the number 81?

We move in the opposite direction, from 81 to 9. The 9 is the square root of 81. The square root is represented by the symbol $\sqrt{\ }$. Thus we write it as

$$\sqrt{81} = 9$$

Can you find another square root of 81 or is 9 the only one?

What is $(-9) \times (-9) = (-9)^2$ equal to?

$$(-9) \times (-9) = 81$$

because minus times minus gives a plus. So -9 is also a square root of 81.
$\sqrt{81} = 9$ or -9, sometimes written as

$$\sqrt{81} = \pm 9$$

where $\pm$ is the plus or minus symbol. The square root of a positive number gives you two numbers, one positive and the other negative.

What is $\sqrt{64}$ equal to?

$$\sqrt{64} = \pm 8 \ (+8 \text{ or } -8)$$

We will generally use the following notation:

$$\sqrt{64} = 8 \text{ (only positive root)}$$
$$\pm\sqrt{64} = \pm 8$$

Most calculators will only give the positive square root. For the other square root we just place a minus sign in front.

In this Introductory chapter when we refer to roots we mean **only** the **real** roots.

We also have the cube root of a number. For example $12^3 = 1728$, so a cube root of 1728 is 12. The symbol for the cube root is $\sqrt[3]{}$. Hence

$$\sqrt[3]{1728} = 12$$

There is **only one** real cube root.

What is the cube root of 27?

A number multiplied by itself three times gives 27:

$$(\text{number}) \times (\text{number}) \times (\text{number}) = 27$$

Use a calculator. There should be a $\sqrt[3]{}$ button on your calculator.

What is the answer?

It's 3, thus

$$\sqrt[3]{27} = 3$$

Remember there is only one cube root. Similarly we can define other roots, for example

$$3^4 = 81$$

What is the 4th root of 81, $\sqrt[4]{81}$, equal to?

$$\sqrt[4]{81} = 3 \text{ because } 3^4 = 81$$

In this case -3 is also a root. So

$$\pm\sqrt[4]{81} = \pm 3$$

In general, the **even** root of a number results in two **numbers**, one positive and the other negative. For **odd** roots we only obtain **one** root.

Example 5

Calculate

a $\pm\sqrt{25}$ **b** $\pm\sqrt{1444}$ **c** $\sqrt{19600}$ **d** $\sqrt[3]{8}$ **e** $\sqrt[3]{-8}$ **f** $\pm\sqrt[4]{625}$

Solution

a Since $5 \times 5 = 25$ and $(-5) \times (-5) = 25$ we have

$$\pm\sqrt{25} = \pm 5 \ (5 \text{ or } -5)$$

b Use the $\sqrt{}$ button on your calculator:

$$\pm\sqrt{1444} = \pm 38$$

c Similarly $\sqrt{19600} = 140$ (only the positive root)

d **What is the cube root of 8, $\sqrt[3]{8}$, equal to?**

We know $2 \times 2 \times 2 = 8$, so $\sqrt[3]{8} = 2$ (only one real root).

e Also $(-2) \times (-2) \times (-2) = -8$. Thus

$$\sqrt[3]{-8} = -2$$

Try this on your calculator by using the $\sqrt[3]{}$ button.